Two-body spectra of pseudoscalar mesons with an $O(a^2)$–improved lattice action using Wilson fermions*

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We extend our calculations with the second-order tree-level and tadpole improved next-nearest-neighbor action to meson-meson systems. Correlation matrices built from interpolating fields representing two pseudoscalar mesons (π–π) with relative momenta $\vec{p}$ are diagonalized, and the mass spectrum is extracted. Link variable fuzzing and operator smearing at both sinks and sources is employed. Calculations are presented for two values of the hopping parameter. The spectrum is used to discuss the residual interaction in the meson-meson system.

The emergence of hadronic forces from first principles is a fundamental question that poses itself to QCD. Remarkably, only few attempts have been made to extract effective interactions, or potentials, between two composite hadrons from the lattice [1]. This task is numerically very challenging since the residual interaction between color singlet composites is about 10^{-2} to 10^{-3} smaller than a typical hadron mass. In this paper we describe an attempt to study the residual interaction in a simple meson-meson system from four-dimensional lattice QCD.

The improved gluonic action used in our study includes planar six-link plaquettes $U_{\rm rt}$ in addition to the elementary plaquettes $U_{\rm pl}$

$$S_{\rm G}[U] = \beta \left[\sum_{\rm pl} (1 - \frac{1}{3}{\rm ReTr}U_{\rm pl}) - \frac{1}{20u_0^2} \sum_{\rm rt} (1 - \frac{1}{3}{\rm ReTr}U_{\rm rt}) \right] . \quad (1)$$

The improved Wilson fermionic action involves nearest-neighbor and next-nearest-neighbor couplings

$$\begin{aligned} S_{\rm F}[\psi,\bar\psi;U] = & \\ & \sum_{x,\mu} \Big\{ \frac{4}{3}\kappa \left[\bar\psi(x)(1-\gamma_\mu)U_\mu(x)\psi(x+\mu) \right. \\ & \left. +\bar\psi(x+\mu)(1+\gamma_\mu)U_\mu^\dagger(x)\psi(x)\right] \\ & -\frac{1}{6}\frac{\kappa}{u_0}\left[\bar\psi(x)(2-\gamma_\mu)U_\mu(x)U_\mu(x+\mu)\psi(x+2\mu) \right. \\ & \left. +\bar\psi(x+2\mu)(2+\gamma_\mu)U_\mu^\dagger(x+\mu)U_\mu^\dagger(x)\psi(x)\right] \Big\} \\ & -\sum_x \bar\psi(x)\psi(x) , \end{aligned} \quad (2)$$

with the hopping parameter κ. Both actions are corrected for discretization errors to $O(a^2)$ at the classical level and contain the tadpole factor u_0 [2,3].

Starting from a π^+-like field where the quarks carry the flavors u and d

$$\phi_{\vec p}(t) = L^{-3}\sum_{\vec x} e^{i\vec p\cdot\vec x}\bar\psi^{\rm d}(\vec x,t)\,\gamma_5\psi^{\rm u}(\vec x,t) \quad (3)$$

we construct a pion-pion operator [4] with relative lattice momentum $\vec p$ and total momentum $\vec P = \vec 0$

$$\Phi_{\vec p}(t) = \phi_{-\vec p}(t)\,\phi_{+\vec p}(t) . \quad (4)$$

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To enhance the overlap of the interpolating field with the ground state we employ link variable fuzzing and operator smearing [5]. In particular, fuzzy links are used to construct the smeared fermionic operators. Smearing is done at both sinks and sources. Quark propagator matrix elements are computed using a random-source technique [4].

The meson four-point time correlation matrix

$$C_{\vec{p}\,\vec{q}}(t,t_0) = \langle \Phi^\dagger_{\vec{p}}(t)\,\Phi_{\vec{q}}(t_0)\rangle - \langle \Phi^\dagger_{\vec{p}}(t)\rangle\langle \Phi_{\vec{q}}(t_0)\rangle \quad (5)$$

consists of a free and an interaction contribution

$$C = \overline{C} + C_{\mathrm{I}} \ . \quad (6)$$

The free two-pion correlator is diagonal

$$\overline{C}_{\vec{p}\,\vec{q}}(t,t_0) = \delta_{\vec{p}\,\vec{q}}\ |c_{\vec{p}}(t,t_0)|^2 \ , \quad (7)$$

where $c_{\vec{p}}$ is the single-meson two-point function. In the present study we are interested in the partial wave $\ell = 0$ contained in the irreducible representation A_1 of the lattice symmetry group $O(3,\mathcal{Z})$. We will identify the corresponding reduced matrix elements by a superscript (A_1).

The eigenvalues of the correlators decrease exponentially with increasing time t

$$\begin{array}{lcll} \overline{C}^{(A_1)}(t,t_0) & \sim & e^{-\overline{E}_p(t-t_0)} & \text{free} \\ C^{(A_1)}(t,t_0) & \sim & e^{-E_n(t-t_0)} & \text{interacting} \ . \end{array} \quad (8)$$

The mass spectra were obtained from linear fits to the logarithm of the eigenvalues of the correlators.

We define an effective interaction through [4]

$$H_{\mathrm{I}} = -\frac{\partial}{\partial t}\ln(\overline{C}^{\,-1/2}\, C\ \ \overline{C}^{\,-1/2}) \ . \quad (9)$$

This yields the matrix elements $(\vec{p}\,|H_{\mathrm{I}}|\vec{q}\,)$. The Fourier transform to coordinate space

$$\begin{aligned} (\vec{r}\,|H_{\mathrm{I}}|\vec{s}\,) &= L^{-3}\sum_{\vec{p}}\sum_{\vec{q}} e^{i\vec{p}\cdot(\vec{r}-\vec{s})}e^{-i\vec{q}\cdot(\vec{r}+\vec{s})} \\ &\qquad (\vec{p}-\vec{q}\,|H_{\mathrm{I}}|\vec{p}+\vec{q}\,) \\ &= \delta_{\vec{r},\vec{s}}\,V_{\mathrm{loc}}(\vec{r}\,) + \text{nonlocal part} \end{aligned} \quad (10)$$

contains a local potential

$$V_{\mathrm{loc}}(\vec{r}\,) = \sum_{\vec{q}} e^{-2i\vec{q}\cdot\vec{r}}(-\vec{q}\,|H_{\mathrm{I}}|+\vec{q}\,) \ , \quad (11)$$

which stems from the $\vec{p}$ independent part of the H_{I} matrix element in (10). The s-wave projection ($\ell = 0$) is

$$\begin{aligned} V_{\mathrm{loc}}(r) &= \frac{1}{4\pi}\int d\Omega_{\vec{r}}\, V_{\mathrm{loc}}(\vec{r}\,) \\ &= \sum_{q^2} j_0(2qr)(q^2|H_{\mathrm{I}}^{(A_1)}|q^2) \ , \end{aligned} \quad (12)$$

where j_0 is a spherical Bessel function.

This preliminary study of the two-pion system was performed on an $L^3 \times T = 9^3 \times 13$ lattice with $\beta = 6.2$ in the quenched approximation. From the string tension the lattice constant is determined to be $a \approx 0.4$fm corresponding to $a^{-1} \approx 500$MeV. Measurements were taken on 48 independent gauge field configurations separated by at least 1024 sweeps. Figures 1 (a) and (b) show the energy spectra for the free and interacting two-pion system according to (8) for $\kappa^{-1} = 6.44$ and $\kappa^{-1} = 6.57$, respectively. Since the critical value is $\kappa_C^{-1} \approx 5.5$ these translate into rather large quark masses. The spectra reveal that the residual interaction lowers the large-momentum levels of the free system, thus indicating attraction at short relative distances.

Figures 1 (c) and (d) display the s-wave projected local potential resulting from (12). We obtain an attractive pion-pion potential with a range of $< 2a$ and a depth of about $1a^{-1}$. A related study [6] with staggered fermions seems to give less attraction. The highest momentum $p = \frac{2\pi}{L}n$, $n = 4$ caused some numerical instability (though no qualitative change) to $V_{\mathrm{loc}}(r)$. We therefore only used $n = 0\ldots 3$ in our calculation of the effective local potential. The solid lines in Figs. 1 (c) and (d) are the results using 48 gauge field configurations. The dotted lines represent the errors from a jackknife analysis with 8 samples of 42 configurations each. The large errors in (c) indicate that the results for $\kappa^{-1} = 6.44$ are still numerically unstable. For the current exploratory analysis no attempts were made to estimate systematic errors.

We expect that a refined analysis, which would include extrapolation to the chiral limit and proper isospin of the π–π interpolating fields, will make comparison to experimental findings possi-

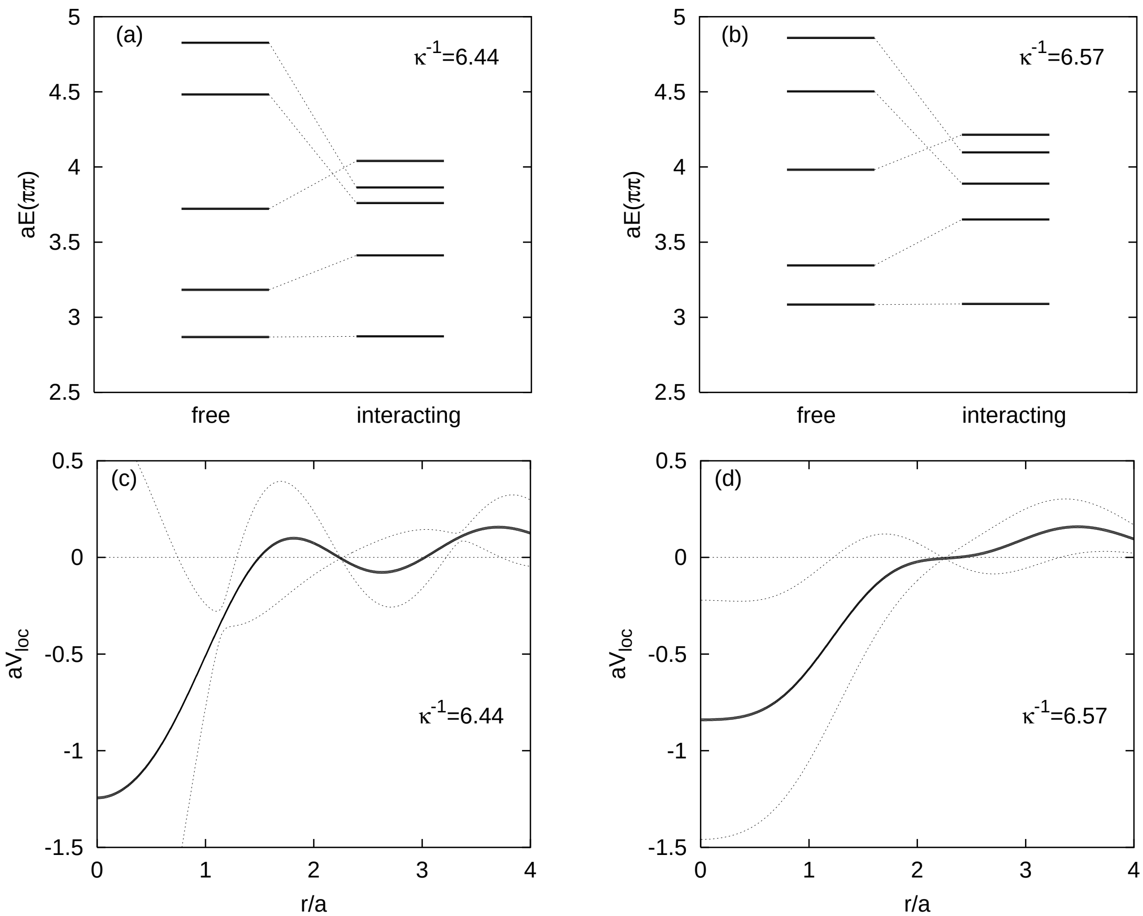

Figure 1. Energy levels for the free and interacting pion-pion system for $\kappa^{-1} = 6.44$ (a) and $\kappa^{-1} = 6.57$ (b) and the corresponding s-wave projected effective local interaction potentials (c) and (d).

ble. Studies similar in spirit to the one initiated here may be extended to various other hadron-hadron systems, like π–N for example.

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